THE PERSUASIVE VALUE OF DEMONSTRATION: DESCARTES' *DISCOURSE*

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Abstract. The aim of this paper is to give an account of the possible doctrines of demonstration which could have served as a mode of exposition in Descartes' *Discourse* and *Essays*. Assuming that Descartes needed to convince his readers that his method was a genuine procedure for advancing scientific knowledge, the presentation of the results in the *Discourse* should have a persuasive role. Granted that the reader would then believe Descartes' method to be an endeavour worthy of attention, the *Dioptrics* and *Meteorology* should also have a demonstrative role. In the first sections, I sketch the early modern disciplines which could provide that, namely syllogism, dialectics and mathematical demonstrations. Next, I suggest that the most plausible source for Descartes' mode of exposition is algebraic geometrical analysis. On this reading, the argumentative value of the suppositions resembles the hypothetical procedure of the analysis. Also, the kind of deduction implied by this procedure is not a formal one, as in the case of the demonstrative syllogism, but rather a relational geometrical one.

Keywords: Descartes, *Discourse on the Method*, demonstration, syllogism, dialectic, geometrical analysis, algebra.

Introduction

In the last two decades, there has been a growing interest in reconstructing Cartesian philosophy starting from Descartes' developments in analytic geometry. Especially since Henk Bos' publication of *Redefining Geometrical Exactness – Descartes' Transformation of the Early Modern Concept of Construction in 2001*,[1] Descartes' early programme of a universal mathematics seems more promising in determining links between his metaphysics and his natural philosophy. For instance, the Cartesian circular definition of *movement* and *body* can be best understood, as Domski argues, considering the account of intelligibility grounded on clear and distinct motions for construction. This kind of motion provides both a basis for physical motion and an acceptable geometrical constuction.[2] My approach will also seek to establish links between Descartes' mathematics and his natural philosophy, but from a different angle: that of his argumentative structure in his *Discourse* and *Essays*. If this type of approach proves to be valid, it will clarify the reasons why Descartes' considered argumentation by way of suppositions an acceptable way of presenting his findings.

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Also, it will better explain the debates with his contemporaries (e.g. Morin's circularity objection) that originated in the usage of this strategy.

The aim of this article is to establish a connection between the mode of exposition used by Descartes in his *Discourse, Dioptrics* and *Meteorology* and his developments of mathematics presented in the *Geometry*. Namely, I will argue that the heuristic procedure of algebraic geometrical analysis is analogous to the argumentative strategy (demonstrating by way of suppositions) in the *Dioptrics* and *Meteorology*.

There are many issues concerning the differences in Descartes' early and late metaphysics. It is clear that his metaphysics is not described in the same way in all of his works. This can mean that he just presents it differently or, more plausible, that he modifies (part of) his philosophical conception.[3] I do not aim to provide an internalist reconstruction of Cartesian metaphysics, so my concern will not be to examine these differences, but merely to present some possible sources of Descartes' expository means and suggest they are influenced by the mathematical procedures he investigated. For this reason, I will refer only to Descartes' early works, the *Rules*, the *Discourse* and *Essays*, and some relevant letters before 1641.

Two things should be carefully distinguished from the beginning. The first is the persuasive property of the *Discourse* and *Essays*, which Descartes hoped to achieve. Its success depends on the conviction it generates within an audience, and its resources are of a rhetorical nature. Thus the type of arguments, their disposition, and the stylistic devices used in the texts belong to this category. The second is the object that Descartes advocated for: his method. The purpose of this method was attaining certainty and expanding knowledge. For it to be considered a genuine procedure of discovery, the method had to have both fertility in advancing knowledge and demonstrative virtues. My claim refers to the first category, as I will try to point out the resources of the *Discourse* and *Essays*' persuasiveness. However, this distinction is not historically accurate, because certain features overlap. For instance, the tradition until the seventeenth century tied the advancement of knowledge to dialectical topics and disposition of arguments, as I will explain in the next sections.

In the first part, I will show how Descartes described his ambition with the *Discourse*. Also, I will provide an account of the possible instances involved in the process of convincing an audience. Next, I will present a historical background of the traditional relation between logic, dialectic and rhetoric. The focus will be on the scope of each discipline and on explaining Descartes' reasons for rejecting the first two as methods of producing true knowledge. I will argue that these are not sufficient to satisfy the instances involved in the act of conviction. By contrast, in the third part I suggest that (classical and algebraic) geometrical analysis is able determine an alternative way of argumentative arrangement. However, this would inherit some problems of the analysis, which I will also detail here. The next part will concentrate on the improvements made by Descartes to algebraic analysis. I will illustrate these by referring to Descartes' solution to the Pappus Problem in the *Geometry*. In the last part, I will return to the Discourse and present the probable sources of its persuasiveness. I will argue that the heuristic of the algebraic analysis accounts for the argumentative strategy which Descartes justified in the *Discourse* and used in the *Dioptrics* and *Meteorology*.

The ambitions of the *Discourse*

Right from the first part of the *Discourse*, Descartes seems very honest about the purpose of exposing his method. He tells his readers that his aim is just to present a personal example, a history of the way he tried to direct his own reason.[4] In doing so, he introduces a type of deliberative discourse, in which his method, among others, would be more or less desirable. While avoiding a kind of prescriptive didactical presentational approach that was typically a scholastic one, Descartes makes use of a stylistic device, the fable, which was very popular in the renaissance, especially from Erasmus onwards.[5] Still, Descartes' method has to have other persuasive resources if it is to be considered an example worthy of imitation. Even more, he seems to believe rhetoric to be a talent rather than a result of study, and successful persuasion should be determined by reason:

> Those with the strongest reasoning and the most skill at ordering their thoughts so as to make them clear and intelligible are always the most persuasive, even if they speak only low Breton and have never learned rhetoric.[6]

It might be argued that these ideals are of a rhetorical nature. The Jesuits of La Flèche laid emphasis on two types of discursive style: the baroque Asiatic and the Attic.[7] The Asiatic style was very affected and emotional, its aim being the virtuosity of the speaker. On the other hand, the Attic style used a simple vocabulary organized in short sentences that gave a more informative character. It is unclear whether Descartes really advocated for one of these types. The above passage seems to suggest that he supported the Attic style, while other instances suggest otherwise. In a letter from February or March 1628 about the literary virtues of his friend, Guez de Balzac, Descartes claims that an equilibrium between a dense and a diffuse style is most desirable, because even if "[…] the expressions full of sense, the richness of the noble reflections sometimes delight the deepest minds, they often tire them due to their style that is too brief and obscure."[8] It is worth noting that this view follows the path of the education he received at La Flèche. The standard textbook of rhetoric Jesuit colleges used at that time was Cypriano Soarez' *De arte rhetorica*. The textbook presented all the main rhetorical doctrines in a clear and simple way. With strong influences from Agricola, Erasmus, Melanchthon, and Ramus, Soarez supported an argumentative style not too crowded with syllogisms and enthymemes.[9] Of course, there is no need to suppose that Descartes believed one type of style to be superior to all others in any case. Each might be suited for a different discursive aim. A more complete understanding of the passage from the *Discourse* lies deeper than stylistic ideals and can be gained if we consider a similar passage from *Rule 2*, in which Descartes criticizes the probabilistic syllogism:

> But whenever two persons make opposite judgments about the same thing, it is certain that at least one of them is mistaken, and neither, it seems, has knowledge. For if the reasoning of one of them were certain

> and evident, he would be able to lay it before the other in such a way as eventually to convince his intellect as well.[10]

The key concepts which should guarantee persuasion are certainty and intelligibility. If an argument would be intelligible and someone would see its certainty, then his only option would be conviction. The only intellectual instances capable of producing certainty are intuition and deduction. Descartes describes them in *Rule 3*.[11] The intuition is presented as a clear and indubitable conception of the mind about an intelligible content. The role of deduction is to preserve the certainty in reasoning. Thus, any process inference has two steps. The first – intuition – provides the clear and distinct impression about some propositional content. The second – deduction – is the act of getting from a premise (or more) to a conclusion via a long chain of inferences. For instance, if A=B, B=C, C=D, D=E, then A=C is an intuition, and the path to E is made through deduction, which memorizes a long string of terms and transfers the intuition all the way to the conclusion. Yet the conclusion, A=E, becomes also an intuition, because it is instantly grasped by the intellect. In this aspect, deduction is reducible to intuition, even if its Cartesian sense is very broad and it could mean anything from explanation, proof, justification to a mere narration of an argument.[12] The intuition rests solely on 'the natural light of reason' which Descartes holds as primitive, as the intellectual perceptions of clarity and distinctness are determined by it. However, the fact that one does obtain clear and distinct ideas does not make them true. The truth must be already out there, an intuition can only recognize it. Furthermore, certainty and conviction are not reversible. Once someone has a clear and distinct intuition, he has to have some degree of certainty.[13] This is the common point of the Cartesian method and the mode of presentation it ought to have. Both have to provide a degree of certainty through the intuitions and the type of deduction used. A viable mode of presentation is not necessarily committed to rules of formal deduction, but rather on the way clear and distinct intuitions can be transferred from the speaker to an audience. What would be the appropriate topics of invention and disposition of such a mode of presentation? Given that the object of Descartes' *Discourse* is a method of expanding knowledge, the relation between this and other models of knowledge must be explained. In the second part, he describes his dissatisfaction about some of the methodologies he knew, which ultimately drove him away from the tradition.[14] Logic had two main problems: it could only be used as a didactical device, not as a method of discovery, and it was mixed with many superfluous concepts. Classical analysis and algebra were only applicable in a narrow field, being very tied to geometrical figures and to rules and symbols. Still, could these disciplines provide a clear and intelligible argumentative arrangement? In the next sections I will explain the extent in which these could give a type of deduction that would reinforce conviction.

Syllogism and dialectics

In the early modern period the art of persuasion was a basic ingredient of college education.[15] The art of persuasion contained logic (appeal to reason) and rhetoric (appeal to imagination and emotions). Around 1611, his fifth year at La

Flèche, Descartes studied rhetoric and in the next year dialectics.[16] Alongside grammar, these disciplines formed the verbal arts, the *trivium*.

For Aristotle, logic was divided into demonstration and dialectics. Demonstration used arguments in which the conclusion was deduced with certainty. On the other hand, dialectics was concerned with arguments leading to a plausible conclusion. Following the medieval tradition, extending knowledge was closely tied to the demonstrative syllogism and on the way in which the conclusion is deduced from the premises. As Larivière explains, knowing that 'C is A' is a direct consequence of knowing that 'B is A' and 'C is B'.[17] Any gained knowledge should follow these steps. The truth of the premises can only be guaranteed by the same kind of deduction, or by the fact that they are considered primitive. 'C is A' is explained by the causal relations of the natures of the terms in the premises. This determines a distinction between the demonstrative syllogism *of fact* (*demonstratio quia*) and demonstrative syllogism *of the reasoned fact* (*demonstratio propter quid*). When the middle term (B) is an accident, we have knowledge without explanation (*demonstratio quia*), while in case the middle term is an essence, we have explanatory knowledge (*demonstratio propter quid*).[18] Here, we are given a cause or an explanation of the conclusion. Thus, the former is somewhat less informative than the second. This is the reason why Aristotle thinks the *demonstratio propter quid* produces understanding, while the *demonstratio quia* does not.

For the case of the demonstrative syllogism, there was the opinion that it could lead to certain knowledge through the method of *regressus*.[19] Zabarella names this procedure a method of discovery. The *regressus* was a procedure that combined *demonstratio quia* and *demonstratio propter quid*. After a *demonstratio quia*, which starts from accidents, not essences, the whole deductive chain was reconstructed as an explanatory demonstration, with the aim of finding the proximate causes which had explanatory properties. Even if we accept that this kind of method produces something in addition to the truth of the premises – not new truths, because the syllogisms are still demonstrative, but understanding and explanation – it cannot be used as a method of research which advances knowledge. This is so because the syllogistic system was devised for a dialectic use between two people, for convincing one about the evidence of a truth. It had a didactical character, with explanatory and mnemonic virtues.[20] Besides the claim that syllogisms cannot advance knowledge, Descartes writes an extensive critique in *Rule 10*:

> […] dialecticians are unable to formulate a syllogism with a true conclusion unless they are already in possession of the substance of the conclusion, i.e. unless they have previous knowledge of the very truth deduced in the syllogism. It is obvious therefore that they themselves can learn nothing new from such forms of reasoning, and hence that ordinary dialectic is of no use whatever to those who wish to investigate the truth of things. Its sole advantage is that it sometimes enables us to explain to others arguments which are already known. It should therefore be transferred from philosophy to rhetoric.[21]

One of the most important (if not the strongest) objection to the syllogism as a resource of knowledge is its circularity. In fact, this is a twofold objection. The first aspect of circularity is the sceptical rejection formal rules of inference. Take the standard example: 'All Men are Mortal / Socrates is a Man // Socrates is a Mortal'. A sceptic would object that it is impossible to determine whether all men are mortal if the fact that Socrates is a man is not already known. The second aspect of circularity is related to the epistemological value of a syllogism. Nothing more will be known in the conclusion than what is already stated in the premises. In a letter to Mersenne from 16 June 1641, Descartes acknowledges the virtues of the of the *demonstratio propter quid*, nevertheless decreasing its applicability:

> I use the word 'idea' to mean everything which can be in our thought, and I distinguish three kinds. Some are adventitious, such as the idea we commonly have of the sun; others are constructed or made up, in which class we can put the idea which the astronomers construct of the sun by their reasoning; and others are innate, such as the idea of God, mind, body, triangle. […] Now if from a constructed idea I were to infer what I explicitly put into it when I was constructing it, I would obviously be begging the question; but it is not the same if I draw out from an innate idea something which was implicitly contained in it but which I did not at first notice in it. Thus I can draw out from the idea of a triangle that its three angles equal two right angles, and from the idea of God that he exists, etc. So far from being a begging of the question, this method of demonstration is even according to Aristotle the most perfect of all, for in it the true definition of a thing occurs as the middle term.[22]

Here, Descartes relates his own demonstration to a *demonstratio potissima*, in which both the cause and the effects are provided. However, this is an option only for innate ideas, such as geometrical entities. If we consider the innate idea of a triangle, the demonstration that three angles equal two right angles is not contained in the innate idea, thus being both informative (the mathematical demonstration) and expanding knowledge (providing additional properties of the triangle). But if we consider the *demonstratio propter quid*, no term can be considered innate, representing 'immutable and eternal essences'. Thus this method is dependent on the nature of the terms in question, not on the mechanics of the argument.

What did the syllogistic system lack in order to provide an argumentative structure? Not the certainty of the deductions, but the circularity of the knowledge. If Descartes were to use the *demonstratio quia* and *propter quid* as the arrangement of his claims, he would invalidate his methods' claim of being capable to produce new truths. He would also invalidate the aim of the *Discourse*, which was not to teach something already known, but to present his own example of guiding his reason.

Scholastics regarded Aristotle's topics as a way of providing new terms for extending knowledge. Due to the fact that these terms were not primitive or categorically deduced, syllogisms of this kind were not demonstrative, their conclusion being more or less plausible. The backbone of the dialectic as a way of advancing knowledge was the search for viable middle terms. Yet, this search had no mechanical way of determining them, like the one a demonstrative syllogism had for establishing a valid inference. For Aristotle, a deduction is dialectical (not demonstrative) when it is based on reputable opinions (*endoxa*).[23] The use dialectic has for Aristotle is in intellectual training, establishing causal links and in the 'philosophical sciences'.[24] Dialectic is thus conceived as a kind of science meant to resolve the differences of opinion by an authoritative appeal to reputable opinions. A dialectical problem was a possible disagreement of reputable opinions, and the opponents had the *ethos* of the opinion which they advocated. The common ground of the opponents was given by the Aristotelian theory of *topics*, which should provide a general methodology for searching the truth. The *commonplaces* (the generally accepted truths) were the core of this methodology.[25] These served as rules for the acceptability of the dialectical arguments.

Cicero greatly modifies the understanding of dialectic. It places dialectic on the same level as logic, both being branches of the *ars bene disserendi*. This art was the genus, the *inventio* (synonym with dialectic – the art of finding arguments) and the logic (the art of evaluating arguments) being the species. Dialectic is no longer conceived as a fundamental science of any dispute. The *commonplaces* also change their meaning from universal rules of debate to mere classifications of arguments.

Cicero's perspective was reinforced by Boethius' *De differentiis topicis*. The later medieval and renaissance developments of rhetoric were undeniably based on Cicero and Boethius' perspective on topics.[26] Boethius proposes that the search for adequate middle terms should be done by appeal to *maximal propositions* and *differentiae*. In a nutshell, maximal proposition are self-evident, unprovable truths. Their function is similar to the topics, to guarantee the validity or the strength of an argument. The *differentiae* should help by suggesting the type of maximal proposition best suited for each argument.[27] By introducing the *differentiae* (mostly inspired by Aristotle's categories), Boethius drops the distinction between the demonstrative and probabilistic syllogism. The topics were used more like a recipe for the classification of arguments rather than a learning tool. Or, if learning meant only memorizing[28] the topics were part of rhetoric and helped memorizing discourses. The focus on memory and on topics as mere classes of arguments was the main transformation of dialectic in the early middle ages.

The change was continued by Rudolph Agricola, who reduced dialectic to logic almost completely. It contained only invention (*inventio*) and arrangement (*dispositio*). Agricola tried to reinvent the topics. His view was that the topics work by helping develop mental connection between propositions or questions.[29] His account was a very original one. Peter Mack gives several examples of techniques and concepts which shifted from rhetoric to logic once *De inventione dialectica* started to gain popularity. The disposition of an argument is one of these. As Mack summarises: "What is left to rhetoric is style and in particular the tropes and figures; what is left to dialectic is the detailed working out of the syllogism and other ways of arranging arguments."[30] Agricola also renews some old techniques, like *similitude*. The purpose of this rhetorical device is to determine the audience to imagine a problem in a certain way, and this is considered a powerful tool against a resilient audience. On his account, a receptive audience can be persuaded by a method of exposition. The exposition has to state the discourse as clearly as possible, without necessarily displaying some rigorous logical connection. If the audience is hostile, the method of argumentation should be used. Here, the deductive connections should be as rigorous as they can be. The problem with Agricola's conception of the topics was that it led to methodological complications. The middle terms for an argument implied the comparison of many categorical propositions. The result was a *porphyrian tree* which was almost impossible to follow in practice.[31]

The reduction of dialectic to logic was completed by Peter Ramus. His *Dialectique* had 262 editions from 1555 to 1655.[32] Ramus thought that Aristotle made a mistake when he tried to devise two separate logics, one for sciences and the other for opinions. The logical vocabulary is thus simplified, as concepts like 'argument', 'principle', 'demonstration' or 'topic' have a similar meaning.[33] These terms designated an amount of knowledge which could not be changed, a finite classification of information with presentational roles. The classification was already done, and the search for a new truth implied only finding its place in this system. Dialectic contained all the possible procedures for doing this, and any piece of knowledge, except grammar and *actio*, tend to be conceived discursively.

How could this model of knowledge serve Descartes in creating intelligibility and certainty about issues of natural philosophy? In *The Search for Truth*, Descartes writes:

> If, for example, I were to ask even Epistemon himself what a man is, and he gave the stock reply of the scholastics, that a man is a 'rational animal', and if, in order to explain these two terms (which are just as obscure as the former), he were to take us further, through all the levels which are called 'metaphysical', we should be dragged into a maze from which it would be impossible to escape. For two other questions arise from this one. First, what is an *animal?* Second, what is *rational?* If, in order to explain what an animal is, he were to reply that it is a 'living and sentient being', that a living being is an 'animate body', and that a body is a 'corporeal substance', you see immediately that the questions, like the branches of a family tree, would rapidly increase and multiply. Quite clearly, the result of all these admirable questions would be pure verbiage, which would elucidate nothing and leave us in our original state of ignorance.[34]

Descartes thought that the *porphyrian* maze was impossible to escape because it unnecessarily increased the number of problems posed by the initial question. In this way, no clear intuition could arise. A clear intuition can be reached by deduction (putting aside the term's vagueness) from certain principles. But for the way a *porphyrian* tree works, this would be impossible, as it would result in tautology. The problem rests on the classification of knowledge, which was complete and predetermined. As such, Descartes would have rather transferred the demonstrative and probabilistic syllogisms to rhetoric, as means of presenting something already known. This way, what remains useful from these domains are just disposition and stylistic devices. Both demonstrative and probabilistic syllogisms fail to produce clear intuitions, and their certainty rests only on the formal rules of inference. Thus, the search for clear intuitions must be made elsewhere. It is mathematics that provided the starting point of this search, as he states in the first part of the *Discourse*.[35] In the next section, I will explain why mathematical demonstrations provided an account of self-evident, certain reasoning and why the methods of demonstration appear obscure and tied to the study of geometrical figures.

Geometrical analysis and algebra

The purpose of this section is to show that geometrical analysis could provide an alternative disposition of arguments than syllogistic. The difference would imply the use of another kind of deduction, distinct from the syllogistic formal rules of inference.

In 1612-1613, Descartes studied the *quadrivium* and was introduced, at La Flèche, to other disciplines that used mathematics, such as astrology, perspective, geodesy, architecture or mechanics.[36] Yet the study of these areas was based on a classification made by Clavius about the disciplines that studied real objects mathematically. The 'pure' abstract mathematical fields were arithmetic and geometry. Descartes refers only to these two.[37] If they could provide a successful argumentative strategy, mathematical reasoning should have the property of preserving certainty.

What was the difference between a syllogistic demonstration and a mathematical demonstration? If there is none, then the objection of circularity would apply to mathematical demonstrations as well. Nicholas Jardine emphasises an important debate at the beginning of the seventeenth century about the status of mathematical demonstrations.[38] The issue was whether mathematical demonstrations were more, less or equally certain than demonstrative syllogisms. This was the echo of an earlier debate in the sixteenth century, the *Quaestio de certitudine mathematicarum*, initiated by Alessandro Piccolomini.[39] The debate took into account whether geometrical demonstrations were a third class of demonstrative syllogism, *demonstratio potissima*. If the *demonstratio quia* proceeded from effect to cause and the *demonstratio propter quid* from cause to effect, the *demonstratio potissima* was thought to be a demonstration in which both the cause and effect of an event could be provided.[40] The issue whether geometrical demonstrations were *demonstrationes potissimae* became unpopular later, mostly because it was hard to argue that geometrical demonstrations deal with any of the four Aristotelian types of causes. Therefore, mathematical demonstrations could have a superior or an inferior degree of certainty than the demonstrative syllogism. It seems that Descartes, as Clavius and Galilei did, supported the former possibility.[41]

Normally, mathematical demonstration had two parts: analysis and synthesis, or *resolutio* and *compositio*, by their Latinised names. A process of *resolutio* is also found in the *regressus* method (which combined the *quia* and *propter quid* demonstrations). The difference is that in the case of the syllogism the *resolutio* has only one deductive step, from cause to effect, while in the mathematical case many auxiliary constructions are needed before reaching the *resolutio*. The analysis was a method used by the ancient geometers for proving theorems and solving problems. Analysis is followed by synthesis, in which the theorem (or the problem) is reconstructed step by step in some other order than that of the analysis. As shown in the *Quaestio de certitudine mathematicarum*, analysis is also important for linking the philosophy of mathematics to logic, as Hintikka emphasised.[42] Besides assuming the conclusion as true, analysis depends on the auxiliary constructions that must be made before the demonstration is devised. No successful demonstration can be made without them. This resembles the logical procedure of natural deduction, where auxiliary assumptions are vital and their number is not always easy to predict. A problem (or theorem) was displayed by its *data*. The configuration which arises after the analysis must be different because, in the end, the deduction must arrive at the same *data*. If the configuration would be the same, then the demonstration would be self-evident or circular.

However, there is a problem with analysis. The procedure cannot be a full-fledged method of discovery because the number of auxiliary constructions needed for a demonstration cannot always be predicted, as Hintikka points out.[43] In this sense, the type and the number of auxiliary constructions can be an indicator for the complexity of a problem. Where there is no need of constructions, geometers speak about 'logical arguments' rather than 'geometrical arguments'.[44] When an auxiliary construction is made in an analysis, it increases the number of instances that must be considered (by increasing the number of possible relations between the geometrical figures). In elementary geometry, this number probably could be predicted, but as the complexity increases, the analysis becomes uncertain. Why does analysis need auxiliary construction? The reason is that the whole method is conceived around the need to consider the interrelations and interdependencies of geometrical entities in a definite configuration.[45] The relevant configuration cannot be the one that is given when a problem is set forth, it has to be enhanced with auxiliary constructions.

The need for auxiliary constructions and the impossibility of being predicted played a role in the early modern debates about status of mathematical demonstrations. For instance, the consideration of auxiliary constructions helped Piccolomini's refutation of geometrical demonstrations being conceived as syllogisms of formal cause.[46] The auxiliary constructions needed to resolve a demonstration were not contained in the premises. Later, the need for auxiliary constructions was partially hid by algebraic approaches like those of Viète or Descartes.[47] Yet algebra does not substitute constructions. Instead of auxiliary entities, algebra considers individual equations, but these equations also start from what is *given* in the problem (or theorem). The problem of the unpredictability of analysis was pointed out by Descartes in *Rule 4*:

> But I have come to think that these writers themselves [Pappus and Diophantus], with a kind of pernicious cunning, later suppressed this mathematics as, notoriously, many inventors are known to have done where their own discoveries were concerned. They may have feared that their method, just because it was so easy and simple, would be depreciated if it were divulged; so to gain our admiration, they may

> have shown us, as the fruits of their method, some barren truths proved by clever arguments, instead of teaching the method itself.[48]

Descartes had justified reasons to think that the method was purposely hidden, considering that Pappus was the only one who describe this method in detail.[49] The way that a problem's result was presented did not appeal to the explicit heuristic of the analysis, even if this would have been the most interesting thing to show. Pappus described the method as a set of doctrines for solving theoretical problems:

> Analysis is the way from what is sought – as if it were admitted – through its concomitants in order to something admitted in synthesis. For in analysis we suppose that which is sought to be already done, and we inquire from what it results, and again what is the antecedent of the latter, until we on our backward way light upon something already known and being first in order.[50]

The purpose of the analysis is obtaining a demonstration and is usually followed by synthesis. Pappus writes that in the synthesis "we suppose that which was reached last in analysis to be already done, and arranging in their natural order as consequents the former antecedents and linking them one with another, we in the end arrive at the construction of the thing sought."[51] Thus, the type of reasoning implied by this procedure is a relational one, and the inferences are determined by the given configuration.

In solving the Pappus Problem in *The Geometry*, Descartes' made use of the *problematical* analysis. It is only used for solving problems. The other type seeks the truth of theorems and is called *theoretical* analysis. There are a few differences between these two kinds analysis. The *theoretical* assumes what is sought as true and, advancing through its consequences as if they were true, arrives at some determined result. If the determined result is true, then the conclusion (the thing sought) is also true. If the result is false, the conclusion is also false. The problematical analysis works in an analogous way, except that the conclusion is not assumed as true, but as already known. I mention the distinction to show the applicability of the hypothetico-deductive method in analysis. One could use it by supposing the truth of the assumption (as in the case of logical natural deduction) or by simply supposing the result to be known. In both cases of the analysis, Pappus claims that the direction of the analysis is opposed to that of the synthesis. What does this direction refer to? Or, more precisely, what do the consequences of a conclusion refer to? Hintikka argues that these do not refer to formal, logical consequences, but rather to a kind of geometrical concomitance.[52] The consequences are not about implications, but about the kind of geometrical auxiliary constructions needed for deducing the conclusion. This should be justified by the relational character of the geometrical demonstrations.

The structure of the analysis consists in three steps. It starts by presenting the *data*. The *analysis proper* contains the heuristic procedure and the auxiliary construction needed. The last is the *resolutio*. The fact that the conclusion is true is made clear in the *resolutio*, where some auxiliary constructions can be established as premises from which the conclusion can be inferred. All the constructions have been made in the *analysis proper*, the *resolutio* just acknowledges the fact that the conclusion was obtained in two ways (from the *data* and from the auxiliary constructions), and deduces the fact that it is true. Next, in the synthesis, the construction is made starting from the *resolutio*. Because the analysis investigates the relations between the geometrical objects, even if some formal deduction could describe the deductive steps of the analysis, it would not exhaust it. Formal deduction could not show the practical importance of the heuristic that must lead to the maximum information which a geometer has to have about a geometrical configuration.[53] Only using this heuristic can a mathematician realize what are the auxiliary constructions needed for a demonstration. This was the aim of Descartes' critique of the analysis: it is too tied to the examination of figures.[54] What was valuable in analysis was just this heuristic, which can be more or less mastered, but cannot be properly presented as a method of preserving certainty.

The case of algebra is different, as it seemed to promise more than that, even if it served the same purposes. From Viète onwards, modern mathematicians begin to use algebra as an analytical tool for solving geometrical problems.[55] Algebra was a powerful method, more problems could be solved and more could be formulated. In 1591, Viète advocated algebra as an alternative analytic method for geometry and arithmetic.[56] His method consisted in reducing the problem to equations, usually with one unknown. It had three steps. The first was *zetetics*, the art of translating a problem into one or more equations. The *posistics* was a procedure for bringing the equations to a standard form. The third one was the *rhetics,* in which geometrical solutions were derived from the equations.[57] The problems were thus reduced to the calculation of symbols, regardless of their geometrical or arithmetical character. Algebra becomes a more formalized method and geometrical figures had a lesser importance. Thus, the deductive construction of the synthesis becomes uncertain and more difficult. What was Descartes' objection to algebra? Not the efficiency of the procedure, which was bigger than the one of classical geometrical analysis. The problem was its obscurity, the fact that algebra tended to be a formal study, without evident connection to the study of geometrical figures. For this reason algebra lacked the intelligibility of the demonstrations. Without intelligibility, this type of deduction could not be translated into a viable argumentative structure. I will next focus on the improvements made by Descartes to this method in the *Geometry*.

Descartes' improvement of algebraic analysis

During the time he wrote the *Rules*, Descartes was so confident in the development that he could bring to the algebraic analysis that he tried setting up the foundations of a *mathesis universalis* capable of solving all mathematical problems. In a letter to Beeckman from 26 March 1619, he wrote:

> I have discovered four remarkable and completely new demonstrations. [...] [W]hat I have found to apply in one case can easily be extended to others. It will thus be possible to solve four times as many problems, and much more difficult ones, than was possible by

> means of ordinary algebra. [...] What I want to produce is not something like Lull's *Ars Brevis,* but rather a completely new science, which would provide a general solution of all possible equations involving any sort of quantity, whether continuous or discrete, each according to its nature. [...] There is, I think, no imaginable problem which cannot be solved, at any rate by such lines as these. I am hoping to demonstrate what sorts of problems can be solved exclusively in this or that way, so that almost nothing in geometry will remain to be discovered.[58]

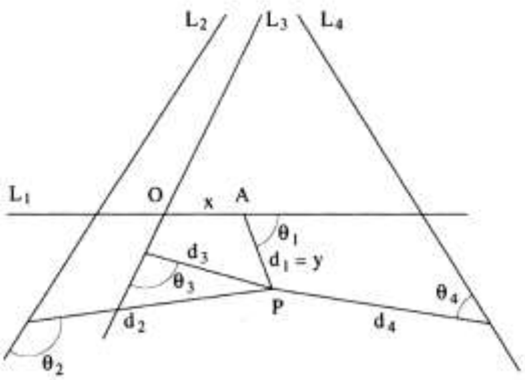
Descartes wasn't satisfied with the application of algebra to geometry. For him, algebra was a method of solving problems. His program of devising a new method for solving any problem rejected the strict demarcation of arithmetic and geometry. This demarcation was based on the fact that arithmetic was dealing with discrete quantities, while geometry with continuous quantities. As long as the problems could be solved without this demarcation, it was no longer, in Descartes eyes, justifiable.[59]

In his mathematical practice, algebra was his main tool. The purpose of his algebraic analysis was reducing the equations to the simplest form. After the analysis, the synthesis contained the calculation of the roots of the equations and the construction was made starting from these roots. He solved the Pappus problem this way. I will reproduce the analysis to follow the steps he had in solving it:[60]

Given data: *n* straight lines L_i in a plane, n angles θ_i and a line segment *a*. For any point *P* in the plane, the oblique distances d_i from the line L_i are defined as the lengths of the segments traced from *P* to L_i, making the angle θ_i with L_i.

Conclusion: Establish the locus of point *P* that must satisfy the ratio δ equal to $\frac{d1d2d3}{ad4d5}$

The figure of the analysis:



Analysis:

1. Assume a coordinate system with its origin at the intersection of *L1* and one of the other lines *(L3* in the figure), its X-axis along *L1* and its ordinate angle equal to *θ1* (with respect to this system *d1 = y)*.

2. By employing the similarity of the relevant triangles, for any point *P* with coordinates *x* and *y,* the corresponding *di* can be written as *di=aix+βiy+γi*, in which the coefficients *ai*, *βi* and *γi* are constant ratios expressed in terms of appropriate constant segments along the *Li* determined by the given position of these lines and the given angles *θi* hence, the *ai*, *βi* and *γi* are known.

3. The constancy of the given ratio *δ* can now be expressed as an equation: *y(a2x+β2y+γ2)...= δ(a)(a4x+β4y+γ4)(a5x+β5y+γ5)*.

The analysis was over when a general irreducible equation was reached. Descartes did not need any auxiliary construction for the analysis. The equations did their job. What remained unchanged was the general procedure. In 1. Descartes assumes a coordinate system with its origin in *O* and *Ox=L1*. This step assumes that the locus of *P* is *already known*, because it is in the same coordinate system as *L1… L4*. So, the locus of *P* can be determined, for the final equation is solvable. Why was this less obscure than the algebraic analysis of Viète? Because he came up with a way in which he could solve arithmetical and geometrical problems in the same way, the aim being to arrive at the most simple equation, which considered knowns and unknowns at the same time. The simplicity of the equation didn't come from its mathematical form, however. It was rather given by the order in which they could be known. In *Rule 6*, Descartes claims that his method should give the possibility of a serial arrangement of objects according to their simplicity. This order did not refer to the ontological status of the objects, but to the status relative to their interdependencies. Any object can be considered *absolute* or *relative*:

> I call 'absolute' whatever has within it the pure and simple nature in question; that is, whatever is viewed as being independent, a cause, single, simple, universal, equal, similar, equal, straight and other qualities of this sort. […] The concept of the relative involves other terms besides, which I call relations: these include dependent, effect, composite, particular.[61]

In this way, starting from clearly intuited premises, one can begin arranging items of knowledge preserving the certainty of the initial intuition. This type of deduction is considering the interrelation between objects of knowledge, from the simplest to the most complex. Thus, certainty can be expanded:

> […] many facts which are not self-evident are known with certainty, provided they are inferred from true and known principles through a continuous and uninterrupted movement of thought in which each individual proposition is clearly intuited.[62]

This kind of deduction seems sufficient for preserving the certainty of the intuitions, provided memory successfully links each individual proposition to basic principles. The interrelations between objects of knowledge could provide such a 'continuous and uninterrupted movement of thought' in the case of solving Pappus' problem. Does this type of arrangement also apply to issues outside of mathematics? In the next section I will argue that this method of demonstration can be the most viable source of exposition for Descartes' natural philosophy in the *Discourse*.

Descartes' use of demonstration methods in the *Discourse*

As he openly states, Descartes did not want to enter deep scholastic debates about natural philosophy. Moreover, it is plausible that his implicit audience was not composed by traditional scholastics.[63] Even so, some of the most important opponents of the *Discourse* were traditional scholastics, like Pierre Bourdin or Jean-Baptiste Morin. Rather, his aim was to persuade people which were not too familiar with Latin and scholastic philosophy. The only exception was the *Geometry*, which required an audience that was familiar (at least) with Euclid's *Elements*. The way Descartes could get their adherence was determined by his method of persuasion, regardless of his appeal to logos or other stylistic devices. In part 5 of the Discourse, he starts to present the natural philosophy from the unpublished *Le Monde*. The conclusions should rest solely on the supposition of God's creation of the world:

> I therefore supposed that God now created, somewhere in imaginary spaces, enough matter to compose such a world […] and that He did nothing but lend his regular concurrence to nature, leaving it to act according to the laws he established.[64]

This is presented as a fable, not as the historical creation of the world. It has the function of a stylistic device which resembles Agricola's *similitude*, meant to make the audience imagine the problem in a certain way, engaging Descartes' autobiographical ethos. This supposition of the creation of the world is of a different nature than the suppositions in the *Essays*. Its maximal persuasiveness is exploited when facing a resilient audience, at least this was the purpose of *similitude*.

Giving the motives of publishing the results of his method in the treatises, he also mentions the existence of a clear foundation, but does not deduce effects from causes as was the traditional fashion:

> I would gladly go on and reveal the whole chain of other truths that I deduced from these first ones. But in order to do this I would have to discuss many questions that are being debated among the learned, and I do not wish to quarrel with them. So it would be better for me, I think, not to do this, and merely to say in general what these questions are, and let those who are wiser decide whether it would be useful for the public to be informed more specifically about them.[65]

What are his persuasive resources if the whole deductive chain is not presented? Daniel Garber believes that presenting by way of suppositions, Descartes made "interesting experiments in exposition."[66] What could their argumentative function be? Even if the argumentation lacks the complete deduction, Descartes seems to be confident in its persuasive value. In part 6, he gives another reason for not presenting the whole deductive sequence, fearing he would be misinterpreted:

> […] but I have deliberately avoided carrying out these deduction in order to prevent certain ingenious persons from taking the opportunity to construct, on what they believe to be my principles, some extravagant philosophy for which I shall be blamed.
> Should anyone be shocked at first by some of the statements I make at the beginning of the Optics and Meteorology because I call them 'suppositions' and do not seem to care about proving them, let him have the patience to read the whole book attentively and I trust he will be satisfied.[67]

From these suppositions, a justified concern about Descartes argumentation arises: it seems circular. The new science that Descartes promised was deductive, but the suppositions of the *Dioptrics* and *Meteorology* are just assumptions, they could not prove anything. It happened in February 1638, when Morin writes to Descartes that his method is circular. In July, Descartes answers, referring to the initial letter:

> You say also that there is a vicious circle in proving effects from a cause, and then proving the cause by the same effects. I agree: but I do not agree that it is circular to explain effects by a cause, and then prove the cause by the effects; because there is a big difference between *proving* and *explaining.* I should add that the word 'demonstrate' can be used to signify either, if it is used according to common usage and not in the technical philosophical sense. I should add also that there is nothing circular in proving a cause by several effects which are independently known, and then proving certain other effects from this cause. I have combined these two senses together on page *76:* 'As my last conclusions are demonstrated by the first, which are their causes, so the first are in turn demonstrated by the last, which are their effects'. But that does not leave me open to the accusation of speaking ambiguously, because I explained what I meant immediately afterwards when I said that experience renders most of these effects quite certain and so the causes from which I deduce them serve not so much to prove them as to explain them – indeed it is the causes which are proved by the effects. And I put 'serve not so much to prove them' rather than 'do not serve at all', so that people could tell that each of these effects could also be proved from this cause, in case there was any doubt about it, provided the cause had already been proved from other effects.[68]

The objection Morin raises is similar to the one about the demonstrative syllogism as being circular. The key feature in understanding this objection is the type of deduction Descartes refers to. If this deduction would be a logical one, the circularity would be real. Yet the arrangement of the claims suggests that this is not the case. Descartes rather induces a certain configuration of elements that need to be considered in their interdependencies. This type of deduction is justified by the fact that the configuration is conceived in a geometrical fashion. The utilisation of suppositions in exposing the results can be a result of the attempt of using algebraic analysis as a method of explanatory presentation. If the explanation was about the same kind of space as the geometrical one, supposing a conclusion and proving through its consequences is perfectly legitimate. With the assumptions of the *Essays*, Descartes followed the same strategy that he did in solving Pappus' problem: he presupposes a system of coordinates on which all objects are placed, as if he already knew that. Then he describes the relations between these objects, exactly like the equations of the problem. Why does he not use the typical order of teaching, from cause to effect, but prefers an order of discovery? This is because he wanted to avoid leading his audience into a traditional scholastic presentation. For this, the heuristic procedure of the analysis better serves as a mode of exposition. The disposition of the arguments is such that the reader could obtain the explanation of some phenomena from the assumed cause and at the same time have that cause proven by other phenomena in an analogous way to the geometrical analysis.

If the audience would arrive at some kind of *resolutio*, by describing the same relations with observable and experimental means, then each supposition would be demonstrated like a geometrical problem. The difficulty is that the number of equations (like auxiliary constructions) cannot be known in advance, and this might be Descartes' reason for not describing all the deductive steps. The algebraic analysis thus remains just a heuristic tool by which he tries to convince the audience of the plausibility of his natural philosophy. Describing the relation between real objects should be sufficient for the continuous and uninterrupted movement of thought that preserves certainty.

Conclusions

Of all the possible sources that Descartes could have used in exposing his first results in natural philosophy, algebraic geometrical analysis best resembles his argumentative structure. Due to the fact that the implicit audience of the *Discourse* was not a scholastic one, Descartes avoided a didactical approach, instead using a more deliberative one. On his account, for an argument to produce conviction, it needs clear and intelligible intuitions and a reasoning that can preserve certainty. The traditional sources of presentation, either demonstrative or dialectical, failed to approximate these argumentative ideals. Algebraic geometrical analysis provided both intelligibility and certainty, but the kind of deduction implied by it was not inferential but relational. The status of the suppositions must be regarded as part of a given configuration, not as a cause from which effects can be formally deduced. On this reading, the dispute with Morin refers to the kind of deduction that supports the *Essays'* claims. The link between Descartes' early natural philosophy and his mathematics can thus be observed at an argumentative level.

Acknowledgements. I want to thank Doina-Cristina Rusu for her encouragements and useful suggestions.

References

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[3] Garber, D. and Cohen, L., "A Point of Order: Analysis, Synthesis and Descartes' Principles", *Archiv für Geschichte der Philosophie* 64/2 (1982): 136-147.
[4] "My present aim, then, is not to teach the method which everyone must follow in order to direct his reason correctly, but only to reveal how I tried to direct my own" Descartes, R., *Œuvres de Descartes* (hereafter AT) eds. Ch. Adam and P. Tannery, 2nd ed. (Paris: Vrin, 1964–1974), 11 vols. I will use the standard translations from *The Philosophical Writings of Descartes* (hereafter CSM) trans. J. Cottingham, R. Stoothoff, and D. Murdoch (Cambridge: Cambridge University Press, 1984–1985), 2 vols. and *The Philosophical Writings of Descartes*, vol. III (hereafter CSMK), trans. J. Cottingham, R. Stoothoff, D. Murdoch, and A. Kenny (Cambridge: Cambridge University Press, 1991). Descartes, R., AT VI, 4; CSM I, 112.
[5] Mack, P., *A History of Renaissance Rhetoric 1380-1620* (Oxford: Oxford University Press, 2011), 312.
[6] Descartes, R., AT VI, 7; CSM I, 114.
[7] Gaukroger, S., *Descartes – An Intellectual Biography* (Oxford: Oxford University Press, 1995), 49-50.
[8] Descartes, R., AT I, 8, my translation.
[9] Mack, P., (2011), 177-182.
[10] Descartes, R., AT X, 363; CSM I, 11.
[11] Descartes, R., AT X, 368; CSM I, 14.
[12] As pointed out by Clarke, D., *Descartes' Philosophy of Science* (Manchester: Manchester University Press, 1982), 64-71.
[13] An interesting argument on the various degrees of certainty that can be generated by a clear and distinct conception of a proposition can be found in Gaukroger, S, *Cartesian Logic: An Essay on Descartes' Conception of inference* (Oxford: Oxford University Press, 2002), 63-67.
[14] Descartes, R., AT VI, 17-18; CSM I, 119-120.
[15] Jardine, N., "Demonstration, Dialectic and Rhetoric in Galileo's Dialogue", in *The Shapes of Knowledge from the Renaissance to the Enlightenment*, eds. R. Kelley and H. Popkin (Dorodecht: Kluwer Academic Publishers, 1991), 105.
[16] The years Descartes spent at La Flèche are debated. Gaukroger, S., (1995), 48-54, believes Descartes studied there between 1606 and 1614.
[17] Larivière, D., A., "Cartesian Method and the Aristotelian-Scholastic Method", *British Journal for the History of Philosophy* 17/3 (2009): 467. 'Consequence' here can mean anything that is conceived through Aristotle's four causes. In Nadler, S., "Doctrines of Explanation in Late Scholasticism and in the Mechanical Philosophy" in *The Cambridge History of the Seventeenth Century Philosophy*, eds. D. Garber and M. Ayers, vol. 1 (Cambridge: Cambridge University Press, 2008), Nadler shows that in the seventeenth century explanations in natural philosophy were almost reduced to efficient causes. However, this was a gradual process, and its peak was not yet reached in the school days of Descartes.
[18] For instance, the same syllogism in the Barbara mode can be a *demonstratio quia* or *propter quid.* Consider the standard example from Aristotle, *Posterior Analytics*, I.13. This is: 1. The Planets do not Twinkle / That which does not Twinkle is Near // The Planets are Near. 2. The Planets are Near / That which is Near does not Twinkle // The Planets do not Twinkle. The first

syllogism is just a *demonstratio quia* because the middle term (that which does not Twinkle) is an accident, while the second syllogism is a *demonstratio propter quid* because the middle term ('that which is Near') is an essence.
[19] For a detailed discussion see Dear, P., "Method and the Study of Nature", in Garber, D. and Ayers, M., (2008), 147-177.
[20] Gaukroger, S., (2002), 20-21, argues that the scholastics mistakenly assimilated the syllogism as a method of discovery, instead of a mode of presentation, as was its initial Aristotelian purpose.
[21] Descartes, R., AT X, 406; CSM I, 36-37.
[22] Descartes, R., AT III, 382-383; CSMK, 183-184.
[23] Larivière, D., A., (2009): 468-471. He believes that the medieval change of *endoxa* (as a respectable opinion) to any opinion is one of the sources of the scholastics' misinterpretation of Aristotle's dialectic. For Aristotle's perspective on *endoxa*, see *The Basic Works of Aristotle*, ed. R. McKeon (New York: Random House, 1941), *Topics* I.1, 100a25-30, 101a35-101b4.
[24] Larivière, D., A., (2009): 468.
[25] Larivière, D., A., (2009): 469-470. However, he suggests that conceiving *commonplaces* as rules of acceptability is debatable, because it does not seem coherent with other passages in Aristotle.
[26] Mack, P., (2011). The first notable exception is, according to Mack, Rudolph Agricola (see chapter 4, 56-76)
[27] Larivière, D., A., (2009): 475-476 gives a very clear example of how do maximal propositions and *differentiae* work in supporting an argument.
[28] As Gaukroger, S., (2002), 6-25, believes.
[29] Mack, P., (2011), 62.
[30] Mack, P., (2011), 61.
[31] Larivière, D., A., (2009): 478, notes that in the worst possible scenario, 784 different comparisons between propositions had to be done.
[32] Larivière, D., A., (2009): 480.
[33] Mack, P., (2011), 143. A clear picture of Ramus' systematization of dialectic can be obtained by consulting the *Tabula Generalis* of Ramus, *Professio Regia*, ed. J. T. Treigius (Berel, 1576), 81.
[34] Descartes, R., AT X, 515-516; CSM II, 410.
[35] Descartes, R., AT VI, 7; CSM I, 114.
[36] Gaukroger, S., (1995), 58.
[37] Descartes, R., AT VI, 7; CSM I, 114.
[38] Jardine, N., "Problems of Knowledge and Action: Epistemology of the Science", in *The Cambridge History of Reinassance Philosophy*, eds. C. Schmitt, Q. Skinner, (Cambridge: Cambridge University Press, 1988) 685-812.
[39] Schöttler, T., "From Causes to Relations: The Emergence of a Non-Aristotelian Concept of Geometrical Proof out of the Quaestio de Certitudine Mathematicarum", *Society and Politics* 6/2 (2012): 29-47. He claims that the debate has not been resolved at that time because of the relational aspect of a geometrical demonstration.
[40] Schöttler, T., (2012): 31
[41] Jardine, N., (1991), 107.
[42] Hintikka, J. and Remes, U., "The Method of Analysis – Its Geometrical Origin and its General Significance", *Boston studies in the Philosophy of Science* 25 (1974): 31-49.
[43] Hintikka, J. and Remes, U., (1974): 31-49.
[44] Hintikka, J. and Remes, U., (1974): 3.
[45] Maybe it is so conceived since the sixteen century, as Schöttler, T., (2012) argues.
[46] For details see Schöttler, T., (2012): 35-36.

[47] Bos, H., J., M., (2001), 145-157.
[48] Descartes, R., AT X, 376-377; CSM I, 19.
[49] Hintikka, J. and Remes, U., (1974): 7.
[50] Thomas, I., *Greek Mathematics* (London: Loeb Classical Library, 1939-1941), vol. 2, 597-601, *apud* Hintikka, J. and Remes, U., (1974): 8.
[51] Hintikka, J. and Remes, U., (1974): 9.
[52] Hintikka, J. and Remes, U., (1974): 12-19. At the end of the paper (appendix 1 and 2) there is a detailed debate about the direction of the analysis and synthesis.
[53] Like the one presented by Hintikka, J. and Remes, U., (1974): 31-41.
[54] Descartes, R., AT VI, 17; CSM I, 119.
[55] Bos, H., J., M., (2001), 9.
[56] Publishing his *In artem analyticen isagoge*: *seorsim excussa ab opere restitutae mathematicae analyseos seu algebra nova* in 1591, Viète opened the way for Descartes' and Fermat's developments of algebra. For details see Bos, H., J., M., (2001), 145-157 and 167-179.
[57] Bos, H., J., M., (2001), 146-147.
[58] Descartes, R., AT X, 154-158; CSMK 2-3.
[59] For a detailed account of Descartes' ideas on demarcation, see Bos, H., J., M., (2001), 352-354.
[60] The analysis is taken from the reconstruction made by Bos, H., J., M (2001), 272-274. It is based on Descartes, R., *The Geometry of Rene Descartes* ed. tr. Smith, D.E., Latham, M.L. (New York: Dover, 1954), 310-314. Bos convincingly argues that the first successful attempts at solving some particular cases of the Pappus problem were made by Descartes after the correspondence with Jacob van Gool (a.k.a. Golius) in 1632.
[61] Descartes, R., AT X, 381-382; CSM I, 21.
[62] Descartes, R., AT X, 369; CSM I, 15.
[63] See a detailed analysis of the rhetorical context of the *Discourse* in Fumaroli, M., *L'âge de l'éloquence: rhétorique et "res literaria" de la Renaissance au seuil de l'époque classique* (Geneva: Droz, 1980)
[64] Descartes, R., AT VI, 42; CSM I, 132.
[65] Descartes, R., AT VI, 40; CSM I, 131.
[66] Garber, D., *Descartes' Metaphysical Physics* (Chicago: University of Chicago Press, 1992), 45.
[67] Descartes, R., AT VI, 76; CSM I, 150.
[68] Descartes, R., AT II, 197-198; CSMK, 106-107.