HUYGENS'S ODD SYMPATHY RECREATED

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Abstract. In 2000, a Georgia Tech research team decided to recreate experiments described by Christiaan Huygens concerning his newly invented pendulum clock. Dormant for 335 years, and resurrected as a historical curiosity, Huygens's system informs and inspires current research in nonlinear science.

Keywords: Christiaan Huygens, pendulum clocks, synchronization, experiments, emergent behavior

1. Introduction

In 1998, the Institute of Mathematics and its Applications (an NSF-sponsored center housed at the University of Minnesota) held a workshop on "Pattern Formation in Continuous and Coupled Systems."[1] The choice of topic reflected the growing importance of this area of nonlinear science. The essential issue is this: how does a disorganized collection of simple entities manage to generate complex organized behavior? Bird flocks provided an everyday example: they display sophisticated flight patterns, including coordinated evasive maneuvers, presumably without benefit of any high-level planning or communication.[2]

One focus of the IMA workshop was a particular type of pattern formation known as spontaneous synchronization. All manner of examples of spontaneous synchronization exist in nature and technology;[3] my talk concerned synchronization of superconducting electronic oscillators. As is customary in technical talks, I began with a brief, broad introduction, starting with an obscure-but-charming story about Christiaan Huygens, which I had picked up from a talk at some previous conference, whose speaker had learned it from still some other talk, in a string that apparently traced back to its mention in the introduction of a book by Blekhman.[4]

The particulars of the story come from a letter written by Christiaan Huygens to his father in 1665, describing a detail he noticed (essentially by accident) while lying ill in bed.[5] It is the earliest known description of spontaneous synchronization, observed in the behavior of two pendulum clocks. For me, this story was a wonderful way to engage an audience before plunging into stupefying technical details. There was, however, one annoying drawback. Rather than ask about superconductors, audiences invariably asked about Huygens's observations, and whether people understood why they synchronized. I didn't, and didn't much care: such an antiquated system hardly seemed like a legitimate research topic. On the other hand, it did seem

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like a fair problem to toss to an undergraduate student, which I did. As it happened, the problem was more subtle than I had first supposed.

2. Background

Synchronization is one of the simplest examples of "emergent behavior." In Huygens letter to his father, he calls the behavior an "odd sympathy" in the motion of two adjacent pendulum clocks. Specifically, the clocks ticked in perfect unison, without deviation as long as they ran. This was astounding because no two clocks have identical periods: one or the other must have the faster pace, and this clock ought to continually outrun the other. But not only did the two clocks tick in lock step, they always swung in "anti-phase," their bobs alternately moving towards each other, then away, then towards, and so on.

Huygens performed a series of experiments in an effort to discover the cause of the sympathy. He was intimately familiar with the physics of single pendulums, and authored a detailed mathematical theory of pendulum motion[6]. Inevitably, Huygens's understanding of the coherent motion of two pendulum clocks was limited: a fundamental theory of mutual synchronization requires developments in nonlinear science made in the latter part of the 20th century. Nevertheless, Huygens was able to determine that the sympathy occurred because of small ("imperceptible") motion of the common support.[7]

Our Georgia Tech team set out to understand the physics behind Huygens's observations. We weren't focussed on making a historically-faithful recreation of his system; rather, we wanted to explain more broadly why synchronization occurs, and under what circumstances. We attacked the problem on two fronts. First, we built an experiment with the same essential ingredients as Huygens's system. Second, we analyzed the mathematical equations governing the motion of the system.

3. Implementation and Experimental Results

Huygens's description of his experiments were written in Latin .[8] We needed a translator, and recruited Heidi Rockwood.[9] The rest of our team were physicists: an undergraduate (Matthew Bennett), an experimentalist (Michael Schatz), and a theorist (myself).

A photograph of our apparatus is shown in Figure 1.[10] The basic elements are as follows. The works of two nominally identical pendulum clocks were mounted side-by-side on a T-shaped frame. The frame was placed on a wheeled cart. The cart rode on a slotted track (running left to right in the photograph) which allowed nearly frictionless motion of the system along the track. Weights could be placed on the cart as a means of systematically varying the experimental conditions. Individually, each pendulum swung with a period of (very nearly) 3/4 second.

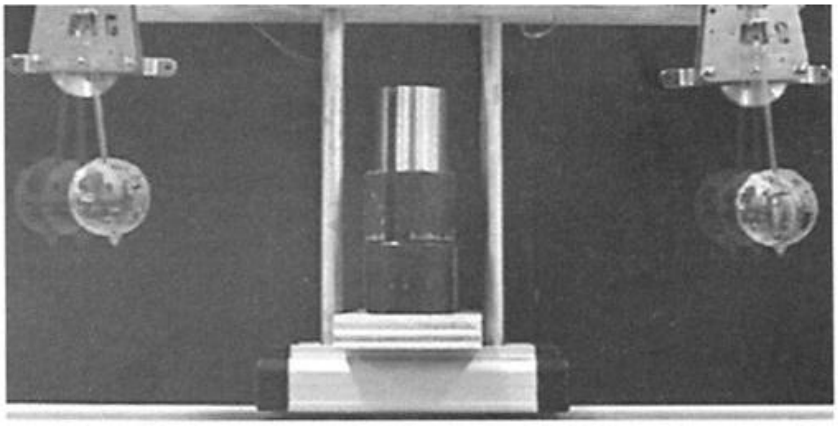

Figure 1: Time lapse photograph of the Georgia Tech apparatus.

Although the primary elements recreate Huygens's system (from the viewpoint of the essential physics), our system differed from his in myriad detailed ways. For example, Huygens's clocks were driven by falling weights, while ours were spring-driven; Huygens's clocks used a verge escapement, while ours used an anchor design; Huygens's clocks were each encased in a box, ours were not; and so on.

Huygens had determined that the coupling between the clocks -- that is, the reason each could affect the other -- was due to small motions of their common support, a heavy wooden beam upon which the clocks were rigidly affixed. From an experimental methodological perspective, our choice for supporting structure had several advantages over a more historically accurate recreation. First, allowing the frame to translate made its motion more reliable and reproducible than using a translationally fixed frame which moved by bending and buckling. Second, placing the frame on a track simplified the translational motion without losing its essential feature (namely, motion parallel to the bobs); this simplification allowed for more accurate measurements. Third, using a wheeled cart virtually eliminated the inherent friction, so that by adding weights we could systematically tune the level of friction, which provided an important control parameter.

Our initial expectation was that the two-clock system would evolve, perhaps slowly, into the "antiphase motion" described by Huygens. But that is not what happened. Instead, in trial after trial, either one or both clocks stopped, well before the clocks ran out of energy. This was completely unexpected -- nothing remotely like it was reported by Huygens. We called this behavior "beating death."

After much playing around, we found that the key to achieving synchronization was to make the individual pendulum periods more nearly identical, as could be done by fine tuning the rod lengths. Once the pendulums synchronized (that is, swung with the same period), their motion further evolved until the bobs swung in anti-phase. We systematically varied the amount of weight added to the cart, did a large number of further trials, and discovered a third possible behavior we called "quasiperiodic:" both clocks kept ticking but ran at different rates, with one clock steadily gaining time on the other. Huygens did describe this non-synchronized behavior in some of his experiments: it is what one expects when the two clocks run independently without significant interaction.

4. Theory, Interpretation of Experiments, and Conclusions

In order to understand our observations, we analyzed the equations governing the motion of the system. Deriving the equations of motion is a straightforward application of Newton's Laws. The resulting equations describe the moment-to-moment evolution of the three dynamical quantities: the angular positions of the two pendulums and the translational position of the cart along the track. In Figure 2, these are denoted by ϕ_1, ϕ_2, and X, respectively. The theoretical model included the mechanisms of energy loss (through friction) and energy gain (provided by each pendulum's escapement mechanism).

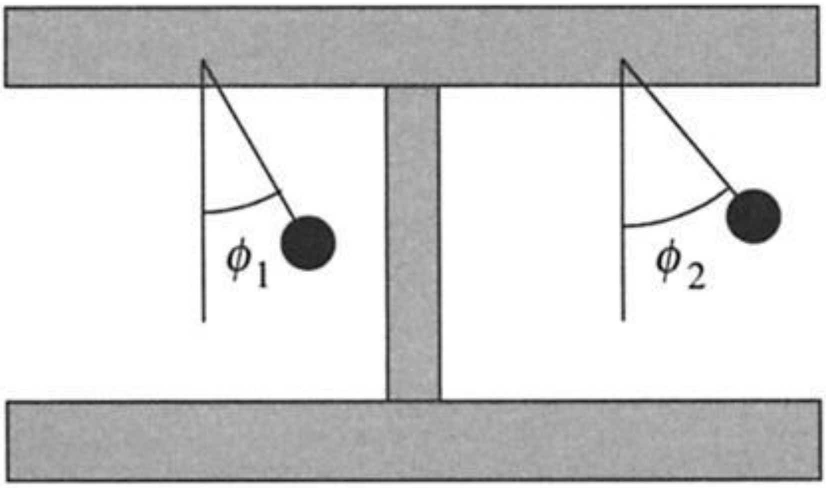

Figure 2: Schematic theoretical representation of the coupled pendulum system.

Although deriving the equations of motion was straightforward, solving them was not, because the system is nonlinear. The classification of a system as linear or nonlinear is a mathematical one. Physically, the distinction is that a linear system can be understood by analyzing each constituent part in isolation: the system-wide behavior is the superposition of the parts. Exploiting this reduction-to-parts makes the mathematical analysis relatively simple. Nonlinear systems cannot be decomposed in this way: feedback between constituent parts can have drastic consequences for the system-wide behavior. Synchronization is an example of intrinsically nonlinear behavior: without nonlinearity, the Huygens's system -- with each clock contributing its own rhythm -- could only show quasiperiodic behavior as described earlier. So, although nonlinearity makes the mathematical analysis difficult, it is also why synchronization is possible at all.

What follows is a summary of the key theoretical findings.[11]

The first thing we deduced from the theory were the key factors determining the system's ultimate fate. These were (1) the mass ratio of one pendulum bob to the total system mass (*i.e.* pendulums plus frame plus cart plus added weights); (2) the friction coefficient between the cart's wheels and the slotted track; and (3) the fractional difference between the pendulums' natural periods (*i.e.* their free-running periods when they sat on the table, uncoupled to each other).

The second thing we learned from the theory was the precise relationship between these parameters and the eventual behavior of the system. This relationship can be summarized by a pair of graphs (see Figure 3). In these graphs, the three key quantities listed above are denoted by μ (the mass ratio), Γ (the friction coefficient), and Δ (the frequency difference).

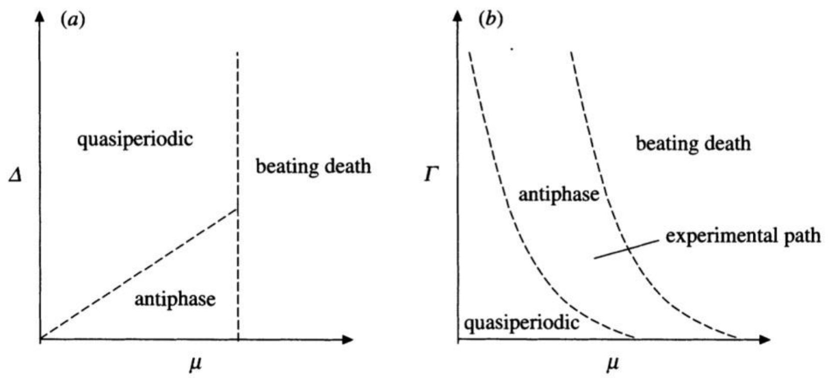

Figure 3: Summary of the theoretical results. There are three distinct behaviors possible: "quasiperiodic" where the two clocks run at different rates, "beating death" where one or both clocks stop running, and "antiphase" where the clocks synchronize (the behavior reported by Huygens).

In fact, the sharp boundary lines shown in these graphs represent a modest simplification of the actual results (both theoretical and experimental) in the following sense. Close to a boundary, the system may settle into either of the corresponding behaviors depending on the initial conditions.

In addition to providing an explanation of the experimental results, the theory gives a more complete picture of what to expect under circumstances not directly tested in the experiments. Recall that the experiments varied the amount of added weight, which directly changes the ratio μ but also slightly changes the friction Γ: the range of conditions tested corresponds to the short segment labeled "experimental path" in Figure 3(*b*). The graphs show what to expect outside this restricted range.

To give an example of how to read Figure 3, imagine a situation where the pendulums' natural periods can be continuously varied, *e.g.* by tuning their lengths. This varies the quantity Δ without affecting either Γ or μ. If Δ is large -- so that the pendulums are decidedly "non-identical" -- the left panel says that the system will settle into one of two states: either (1) the pendulums will run at different frequencies ("quasiperiodic"), if the mass ratio μ is small, or (2) the clock(s) will stop, if the mass ratio μ is large. In the former case, if we then decrease Δ enough, the system will make the transition to antiphase motion. But if the mass ratio is too large (*e.g.* too much added weight), even perfectly matched pendulums ($\Delta = 0$) will not synchronize: only beating death is possible.

Figure 3 represents the scientific goal we had sought, providing a general understanding of what to expect under a variety of circumstances. In addition, as a happy by-product, these results lead us to a curious conclusion about Huygens's observations. Apparently, the fact that his two clocks displayed "sympathy" at all rested on two pieces of good fortune. First, because he intended that the clocks were to be used at sea, Huygens loaded each clock box with roughly 100 pounds of lead in order to keep them upright in stormy seas. If they had not been so weighted, the mass ratio μ would have been too large, and the synchronized state would have been inaccessible. But the extra weighting was not enough: the clocks would not have synchronized if they had not been exquisitely matched (small Δ) and their operation rock solid stable over long periods of time. This technical precision was achieved only as a result of Huygens's inventive clock design and Severyn Oosterwijck's deft craftsmanship.[12]

5. Afterword

We published our findings in the Proceedings of the Royal Society as a matter of symmetry, owing to the historical context of Huygens's communications and collaboration with the newly established Royal Society. Immediately after our paper's appearance, we learned two interesting things.

First, an email from the editor of the Horological Journal[13] informed us that the phenomenon we had called "beating death" was well-known to horologists,[14] at least in the context of a single clock powered by a falling weight. In this phenomenon, variously called "Wednesday stopping" or the "Thursday effect" among other things, the falling weight plays the role of the second pendulum in our experiments. The name comes from the traditional practice of winding tall clocks on Sunday, with enough juice to run a full eight days, only to find it stopped mid-week.[15]

Second, a paper appeared describing a system similar to ours, yet reporting entirely different results.[16] The system comprised two metronomes sitting on a light wooden plank, which rests in turn on two empty soda cans (see Figure 4). The metronomes almost always settled into *in-phase* behavior, and only under exceptional circumstances displayed anti-phase behavior. The author concluded that his results differed from Huygens's (and ours) because the metronomes swung through much large angles (45 degrees, as compared to Huygens's 25 degrees and our 8 degrees).

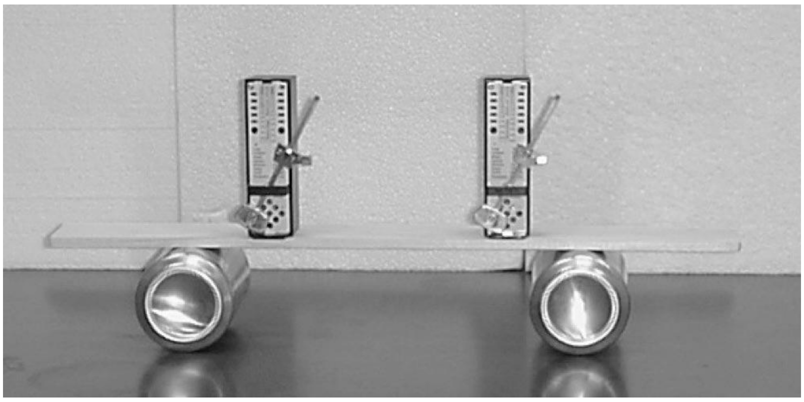

Figure 4: Photograph of a Huygens-like experiment using metronomes instead of clocks (see note 16)

Since the appearance of our paper, there has developed a growing body of technical research on these systems. Indeed, a recent issue of European Physics Journal Special Topics was devoted entirely to the subject.[17] What had seemed like an unassumingly simple, hopelessly ancient, "too low tech for prime time" system is now recognized as a legitimate focus of modern research.

References

[1] Golubitsky, M., D. Luss, and S. H. Strogatz (eds.), *Pattern Formation in Continuous and Coupled Systems* (New York: Springer, 1999).

[2] (A spectacular example can be found here: *Falcon Attack: Peregrine Divebombs Flock of Starlings*, www.youtube.com/watch?v=b8eZJnbDHIg, accessed Nov.15, 2017.

[3] Strogatz, S. H., *Sync: How Order Emerges from Chaos in the Universe, Nature, and Daily Life* (New York: Hyperion, 2003).

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[5] Huygens, C., *Oeuvres completes de Christiaan Huygens*, 22 vols. (The Hague: Martinus Nijhoff, 1893), vol. 5, 241-262. .

[6] Huygens, C., *Christiaan Huygens's the pendulum clock or geometrical demonstrations concerning the motion of pendula as applied to clocks*, trans. R. Blackwell (Ames, IA: Iowa State University Press, 1986).

[7] Huygens, C., *Oeuvres completes*, vol. 5, 241-262.

[8] Huygens, C., *Oeuvres completes.*

[9] Professor Rockwood was, ironically, chairman of Georgia Tech's Modern Languages department.

[10] Full details are provided in Bennett, M., M F. Schatz, H. Rockwood, and K. Wiesenfeld, "Huygens's clocks," *Proceedings of the Royal Society of London* A, volume 458 (2002): 563-579 (2002).

[11] Bennett, M., M F. Schatz, H. Rockwood, and K. Wiesenfeld, "Huygens's clocks," (2002): 563-579.

[12] Prior to Huygens's inventions, typical clocks varied by about "15" minutes per day; a well-adjusted pendulum clock of the 1660s varied only about 15 seconds per day.

[13] Treffry, T., private communication.

[14] Penman, L., *The Clock Repairer's Handbook* (Newton Abbot: David & Charles, 2000).

[15] In preparing this paper, I came across a Royal Society document reporting this behavior in the two-clock context: Ellicott, J., "An Account of the Influence Which Two Pendulum Clocks Were Observed to Have upon Each Other," *Philosophical Transactions* 41 (1739):126-135.
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[17] Kapitaniak, T. and J. Kurths, (eds.),"Dynamics of pendula and their applications," special issue of the *European Physics Journal Special Topics* 223/ 4 (2014).